Bayesian Inference with Probabilistic Programs

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Overview

Aim Explain research in Multiple Target Tracking

Aim Mention research in Programming Langauges

Theme Probabilistic Programs

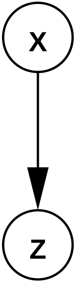
Deterministic programs

$X = 2$
$Z = X + 1$
output Z

Probabilistic programs

$$X \sim \mathrm{unif}(0, 1)$$
$$Z \sim \mathrm{unif}(0, X)$$
$$\textbf{output} \;\; Z$$

Probabilistic programs



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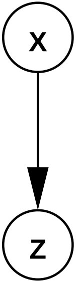
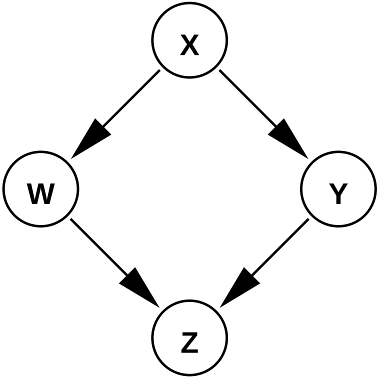
Probabilistic programs



$$X \sim \text{unif}(0, 1)$$
$$Z \sim \text{unif}(0, X)$$
$$\textbf{output} \;\; Z$$

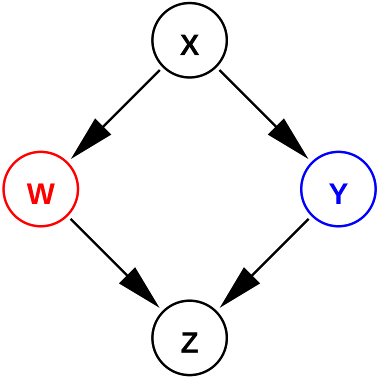
$$P(X, Z) = P(X) \;\; P(Z \mid X)$$

Bayesian networks use definitions



$$X \sim \phi_X$$
$$W \sim \phi_W(X)$$
$$Y \sim \phi_Y(X)$$
$$Z \sim \phi_Z(W, Y)$$
output Z

Bayesian networks use definitions



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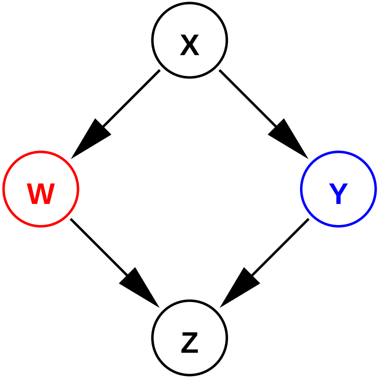

$$X \sim \phi_X$$
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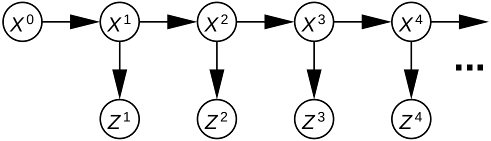
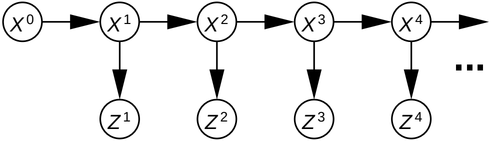
$$Z \sim \phi_Z(W, Y)$$
output Z

HMMs, Bayes filters use loops



$\mathrm{X}^0 \sim \phi_{\mathrm{prior}}$
for t **in** $[1, \ldots]$:
  $\mathrm{X}^t \sim \phi_{\mathrm{trans}}(\mathrm{X}^{t-1})$
  $\mathrm{Z}^t \sim \phi_{\mathrm{obs}}(\mathrm{X}^t)$
  output Z^t

HMMs, Bayes filters use loops



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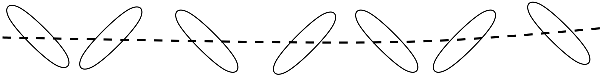
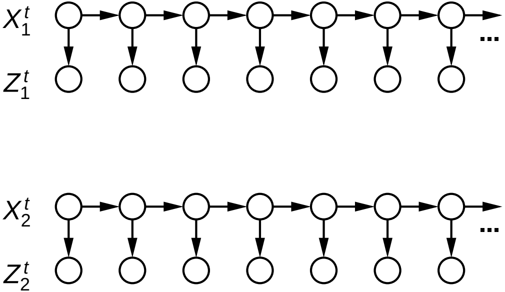
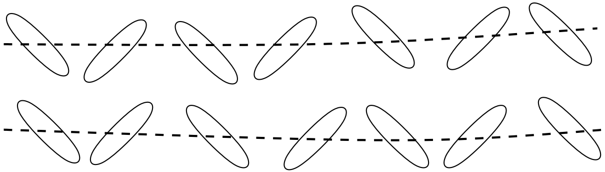


Multiple Target Tracking



$X_1^0 \sim \phi_{\text{prior}}$
$X_2^0 \sim \phi_{\text{prior}}$
for t **in** $[1, \ldots]$:
    $X_1^t \sim \phi_{\text{trans}}(X_1^{t-1})$
    $X_2^t \sim \phi_{\text{trans}}(X_2^{t-1})$
    $Z_1^t \sim \phi_{\text{obs}}(X_1^t)$
    $Z_2^t \sim \phi_{\text{obs}}(X_2^t)$
    output Z_1^t, Z_2^t



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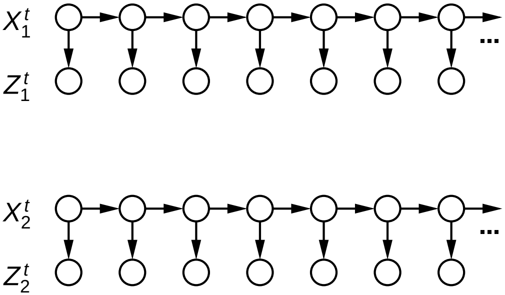
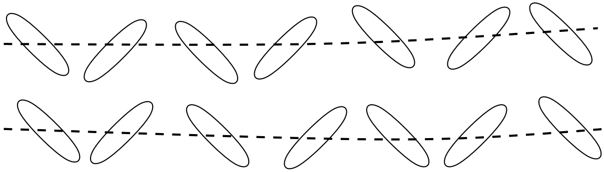
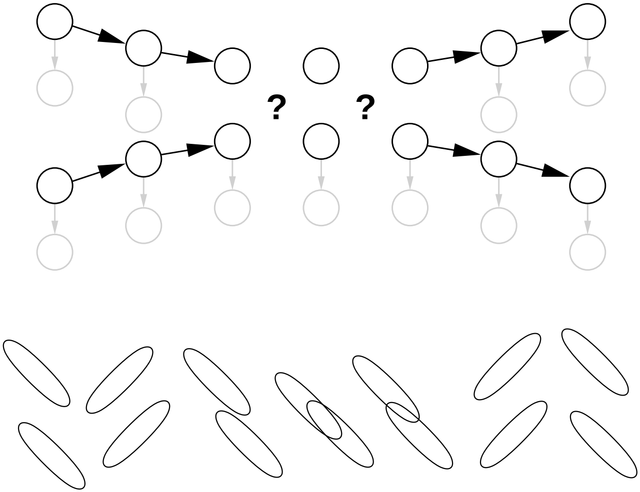


$X_1^0 \sim \phi_{\text{prior}}$
$X_2^0 \sim \phi_{\text{prior}}$
for t **in** $[1, \ldots]$:
  $X_1^t \sim \phi_{\text{trans}}(X_1^{t-1})$
  $X_2^t \sim \phi_{\text{trans}}(X_2^{t-1})$
  $Z_1^t \sim \phi_{\text{obs}}(X_1^t)$
  $Z_2^t \sim \phi_{\text{obs}}(X_2^t)$
  output $Z_?^t, Z_?^t$



...but sometimes association is ambiguous



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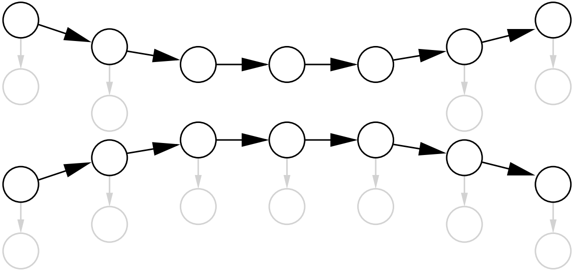

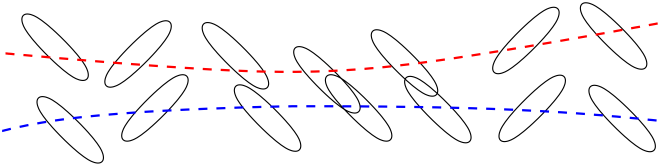

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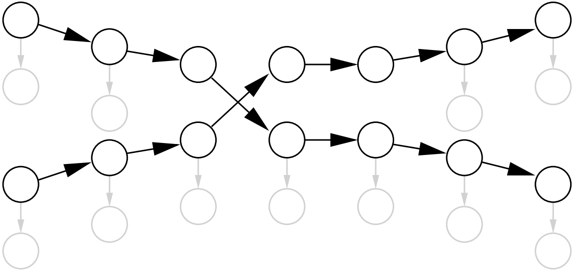

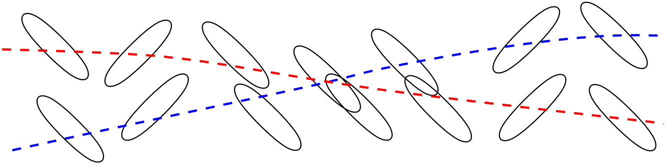

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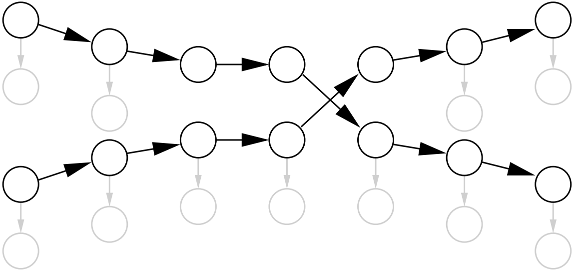
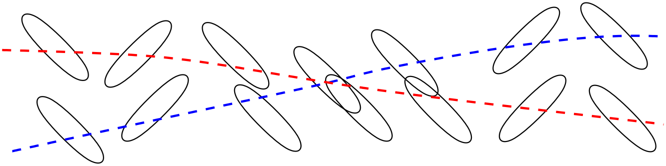
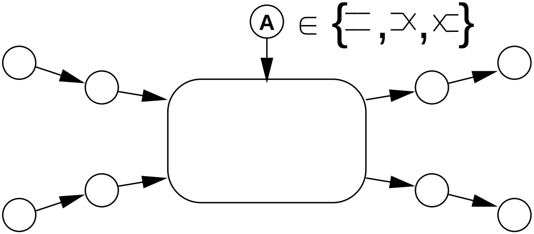




Multiple Hypothesis Tracking



...

for t **in** [1, ...]:

  $A \sim \phi_{\text{assoc}}(\ldots)$

  branch on A:

    case $=$:

      $X_1^t \sim \phi_{\text{trans}}(X_1^{t-1})$

      $X_2^t \sim \phi_{\text{trans}}(X_2^{t-1})$

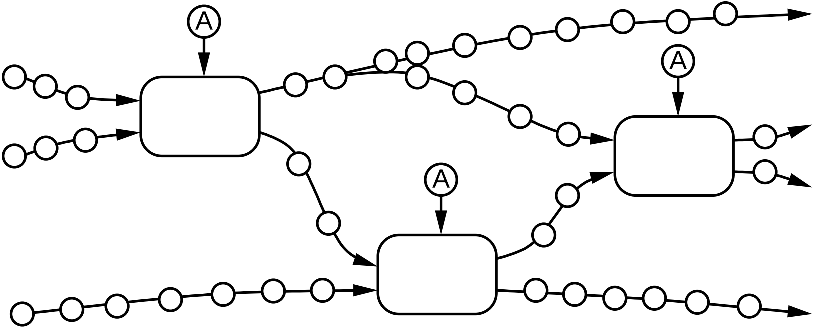
    case $\times$:

      $X_1^t \sim \phi_{\text{trans}}(X_2^{t-1})$

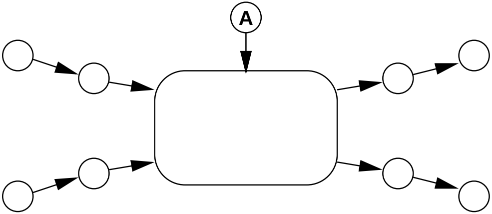
      $X_2^t \sim \phi_{\text{trans}}(X_1^{t-1})$

...

Long-term MHT is too expensive



Bayes Net Tracking Database



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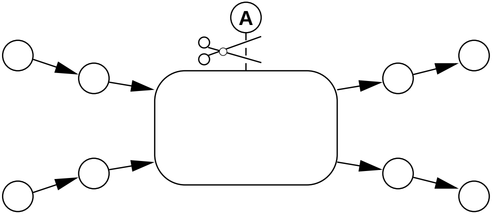

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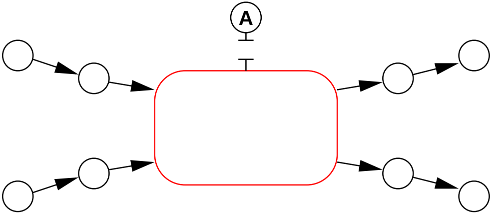


$\ldots$

for t **in** $[1, \ldots]$:

  if X_1^t is close to X_2^t:

    $\underline{X}^t \sim \phi_{\text{mix}}(\underline{X}^{t-1})$

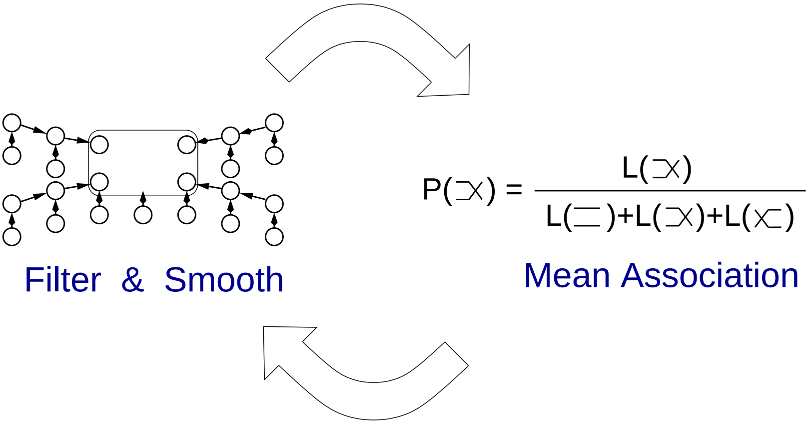
  else:

    $X_1^t \sim \phi_{\text{trans}}(X_1^{t-1})$

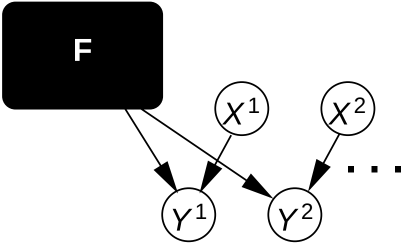
    $X_2^t \sim \phi_{\text{trans}}(X_2^{t-1})$

  $\ldots$

Iteratively compute state+association



What about more general programs?



$F \sim \phi_{\text{program}}$
for i **in** $[1, \ldots]$:
  input X^i
  $Y^i \sim F(X^i)$
  output Y^i

What is a random program?

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$$\begin{array}{rcl} \text{prog} & ::= & \text{S} \\ & | & \text{K} \\ & | & \text{prog(prog)} \end{array}$$

What is a random program?

$$\begin{aligned}\text{prog} \quad ::= \quad & \text{S} \\ | \quad & \text{K} \\ | \quad & \text{prog(prog)}\end{aligned}$$

$$\begin{aligned}\phi_{\text{prog}} = & \ 1/4 \ (\textbf{return} \ \ \text{S}) \\ & + 1/4 \ (\textbf{return} \ \ \text{K}) \\ & + 1/2 \ (\ \ \text{F} \sim \phi_{\text{prog}} \\ & \qquad\quad \text{X} \sim \phi_{\text{prog}} \\ & \qquad\quad \textbf{return} \ \ \text{F(X)} \ \ )\end{aligned}$$

Research in programming languages

Developed tools for coarse equivalence

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Research in programming languages

- Developed tools for coarse equivalence
- Simulated types in untyped languages
- Implemented an equational theorem prover
    - Finds first 10,000 programs-mod-equivalence
    - Generates conjectures from evidence
    - Optimizes language to fit examples

Questions?